

Balogh Conference Schedule Friday

- 9:00 - 12:00 Gathering and registration – 115 Bachelor Hall – coffee, juice, snacks
- 1:00 Registration
- Friday afternoon session in **Room 110** – Bachelor Hall.
- 2:00 - 2:35 Dave Lutzer
Trees and lines
- 2:40 - 3:00 Mike Reed
Computational models of topological spaces
- 3:05 - 3:40 Peter Nyikos
Balogh's 'lost' theorems on paracompactness in locally compact spaces
- 3:40 Coffee
- 3:55 - 4:20 Alan Dow
Radially Compact spaces
- 4:25 - 4:50 Todd Eisworth
Proper forcing revisited
- 4:55 - 5:15 Paul Szeptycki
FUF and boundedly FUF spaces
- 5:20 - 5:40 Mikhail Matveev
Fat-Psi constructions
- 5:45 Dinner – on your own. The food court in Shriver Center should be open.
- 6:30 - ? Party – 115 Bachelor Hall
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Saturday

- 8:00 Coffee, juice, snacks in Room 115 Bachelor Hall
- Saturday morning and afternoon sessions in **Room 102** – Bachelor Hall
- 9:00 - 9:30 Mary Ellen Rudin - BUCKINGHAM SCHOLAR LECTURE
A joint paper
- 9:35 - 10:10 Bill Fleissner
Coarser Connected Topologies
- 10:15 - 10:35 Hal Bennett
Metrizably Fibered GO-spaces
- 10:35 Coffee

10:50 - 11:20	István Juhász <i>Calibers and Tightness</i>
11:25 - 11:55	Jerry Vaughan <i>The Urysohn Property and two step iteration of almost disjoint families</i>
12:00 - 12:20	Frank Tall <i>Riffs on Zoli's "Locally nice spaces under Martin's axiom"</i>
12:20	Lunch
2:00 - 2:20	Strashimir Popvassilev <i>Base-cover and base-family paracompactness</i>
2:25 - 2:45	John Porter <i>An α-normal non-β-normal Moore space</i>
2:50 - 3:10	Prem Sharma <i>Optimal constant weight binary codes</i>
3:10	Coffee
3:25 - 3:50	Dikran Dikranjan <i>Forcing hereditarily separable countably compact group topologies</i>
3:55 - 4:15	Lew Ludwig <i>κ-Fréchet-Urysohn Spaces</i>
4:20 - 4:40	Heikki Junnila <i>Hereditarily normal compactifications of metrizable spaces</i>
4:45	Coffee
5:00 - 5:50	Gary Gruenhagen - BUCKINGHAM SCHOLAR LECTURE <i>The mathematics of Zoltan T. Balogh</i>
6:15 - 7:00	Social Gathering in Towers Room (336 Shriver Center)
7:00	Banquet in Towers Room

The organizers of the ZOLTAN BALOGH MEMORIAL TOPOLOGY CONFERENCE would like to thank several groups for the emotional and financial support which made this conference possible.

Participants of the Zoltan Balogh Memorial Conference
 Participants in the Contributed Problems Section
 Department of Mathematics and Statistics, Miami University
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A NATURAL DOWKER SPACE

ZOLTAN T. BALOGH

ABSTRACT. We will describe a plain version of a recursive technique to construct Dowker spaces.

1. Constructing the space.

Let $X = \mathfrak{c} \times \omega$ and, for all $n \in \omega$, $\mathbb{W}_n = \mathfrak{c} \times n$. The collection $\mathcal{B}_0 = \{\mathbb{W}_n : n \in \omega\} \cup \{X \setminus \{x\} : x \in X\}$ is a subbase for an initial (T_1 , not Hausdorff) topology. We will add more open sets in $2^{\mathfrak{c}}$ steps to \mathcal{B}_0 to make X normal but not countably paracompact.

For $U^0, U^1 \subset X$ we say $S = \langle U^0, U^1 \rangle$ is a *covering pair* if $U^0 \cup U^1 = X$. Fix a list $\mathcal{S} = \langle S_\xi \rangle_{\xi < 2^{\mathfrak{c}}}$ of all covering pairs mentioning each $2^{\mathfrak{c}}$ many times. S_ξ will denote the covering pair $\langle U_\xi^0, U_\xi^1 \rangle$.

Inductively, we will define $H \subset 2^{\mathfrak{c}}$ and, for all $\xi \in H$, a pair $\langle B_\xi^0, B_\xi^1 \rangle$ such that $B_\xi^0 \subset U_\xi^0, B_\xi^1 \subset U_\xi^1, B_\xi^0 \cap B_\xi^1 = \emptyset$ (and $B_\xi^0 \cup B_\xi^1 = X$). Then we'll set

$$\mathcal{B}_\xi = \mathcal{B}_0 \cup \{B_\eta^0, B_\eta^1 : \eta \in H, \eta < \xi\}.$$

For a set to be ξ -open, we mean open in the topology generated by \mathcal{B}_ξ as a subbase. If $A \in [X]^\omega$ and $S = \langle U^0, U^1 \rangle$ is a covering pair then the pair restricted to A is given by $S \upharpoonright A = \langle U^0 \cap A, U^1 \cap A \rangle$. We let $\mathcal{S}_A = \{S \upharpoonright A : S \text{ is a covering pair}\}$.

Definition. $\langle A, \mathcal{R}, u \rangle$ is a *control triple* if $A \in [X]^\omega$, $\mathcal{R} \in [\mathcal{S}_A]^\omega$ and u is a countably infinite function with $\text{dom}(u) \subset A$ and

- (C₁) $x \in \text{dom}(u) \implies u(x) \in [\mathcal{S}_A \setminus \mathcal{R}]^{<\omega}$;
- (C₂) $x \neq x'$ in $\text{dom}(u) \implies u(x) \cap u(x') = \emptyset$.

Let $\langle A_\beta, \mathcal{R}_\beta, u_\beta \rangle_{\beta < \mathfrak{c}}$ be a list of all control triples mentioning each \mathfrak{c} many times and where $A_\beta \subset \beta \times \omega$.

Suppose $\xi < 2^{\mathfrak{c}}$ and for every $\eta < \xi$ we have already decided whether $\eta \in H$ and, if so, what B_η^0, B_η^1 are. We now decide if $\xi \in H$ and, if so, show how to define B_ξ^0, B_ξ^1 .

Case 1. Suppose U_ξ^0, U_ξ^1 are ξ -open and there is no $\eta < \xi$ such that $\langle U_\eta^0, U_\eta^1 \rangle = \langle U_\xi^0, U_\xi^1 \rangle$ and U_η^0, U_η^1 are η -open. Then $\xi \in H$ and we need to define B_ξ^0, B_ξ^1 .

Suppose $\beta < \mathfrak{c}$ and for every $\alpha < \beta, \forall k \in \omega$, we already decided $\langle \alpha, k \rangle(\xi)$ equals the unique $i \in \{0, 1\}$ with $\langle \alpha, k \rangle \in B_\xi^i$.

Subcase 1.1. If $S_\xi \upharpoonright A_\beta \in \mathcal{R}_\beta$, then take the biggest $m \leq \omega$ such that there exists $i \in \{0, 1\}$ with $\{\beta\} \times m \subset U_\xi^i$. Fix an i with $\{\beta\} \times m \subset U_\xi^i$ and make sure $\{\beta\} \times m \subset B_\xi^i$. If this $m < \omega$, then, for $\omega > k \geq m$, pick any $i \in \{0, 1\}$ such that $\langle \beta, k \rangle \in U_\xi^i$ and set $\langle \beta, k \rangle \in B_\xi^i$.

Subcase 1.2. Suppose there exists $x \in \text{dom}(u_\beta)$ with $S_\xi \upharpoonright A_\beta \in u_\beta(x)$. Note that $S_\xi \upharpoonright A_\beta \notin \mathcal{R}_\beta$ by (C₁) and there is only one such x by (C₂). Since $x \in A_\beta \subset \beta \times \omega$, $x(\xi)$ has been defined. For every $j \in \omega$, set $\langle \beta, j \rangle(\xi) = x(\xi)$ unless $\langle \beta, j \rangle \notin U_\xi^{x(\xi)}$ in which case put up with $\langle \beta, j \rangle \in B_\xi^{1-x(\xi)}$.

Subcase 1.3. Neither Subcase 1.1 nor Subcase 1.2. Then for every $j \in \omega$ pick one $i \in \{0, 1\}$ with $\langle \beta, j \rangle \in U_\xi^i$ and set $\langle \beta, j \rangle \in B_\xi^i$.

Case 2. Not Case 1. Then $\xi \notin H$ and B_ξ^0, B_ξ^1 need not be defined.

The space $X = \mathfrak{c} \times \omega$ with

$$\mathcal{B} = \cup_{\xi < 2^{\mathfrak{c}}} \mathcal{B}_\xi = \mathcal{B}_0 \cup \{B_\xi^0, B_\xi^1 : \xi \in H\}$$

Key words and phrases. Dowker space, normal, countably paracompact, elementary submodel.

as a subbase for the topology is normal by construction. We need to prove that X is **not** countably paracompact.

2. Complete neighborhoods.

Each basic open neighborhood for $x = \langle \alpha, k \rangle \in X$ is of the form

$$V_{t,K}(x) = \bigcap_{\xi \in t} B_\xi^{x(\xi)} \cap (\mathbb{W}_{k+1} \setminus K)$$

for some choice of $t \in [H]^{<\omega}$, $K \in [X \setminus \{x\}]^{<\omega}$.

Let us call $V_{t,K}(x)$ *complete* if for every $\xi \in t$,

$$V_{t \cap \xi, K}(x) \subset U_\xi^{x(\xi)}.$$

Lemma 2.1. *For every neighborhood $V_{t,K}(x)$, there are $t^* \supset t$, $K^* \supset K$ such that $V_{t^*, K^*}(x)$ is a complete neighborhood.*

Proof. For every incomplete neighborhood $V_{t', K'}(x)$ let $\xi_{t', K'}$ be the largest $\xi \in t'$ with

$$V_{t' \cap \xi, K'}(x) \not\subset U_\xi^{x(\xi)}.$$

Claim. For every incomplete neighborhood $V_{t', K'}(x)$ there is a neighborhood $V_{t'', K''}(x) \subset V_{t', K'}(x)$ such that either $V_{t'', K''}(x)$ is complete or $\xi_{t'', K''} < \xi_{t', K'}$.

Proof of Claim. Let $\eta = \xi_{t', K'}$. Since $U_\eta^{x(\eta)}$ is η -open there exist $\bar{t} \in [H \cap \eta]^{<\omega}$, $\bar{K} \in [X]^{<\omega}$ with $V_{\bar{t}, \bar{K}}(x) \subset U_\eta^{x(\eta)}$. Then $t'' = t \cup \bar{t}$, $K'' = K \cup \bar{K}$ is as required.

The proof of the Lemma now follows because otherwise we could construct an infinite decreasing sequence: $\xi_{t, K} > \xi_{t_1, K_1} > \xi_{t_2, K_2} > \dots$ \square

3. Homogeneity: Finding and reflecting β .

For $\beta \in \mathfrak{c}$, define $C_\beta = \{\beta\} \times \omega$. We say β is ξ -homogeneous if there exists $i \in \{0, 1\}$ such that $C_\beta \subset B_\xi^i$.

Main Lemma 3.1. *If H' is any countable subset of H then there is some $\beta \in \mathfrak{c}$ such that β is ξ -homogeneous for every $\xi \in H'$.*

The proof of the Main Lemma will follow after the proofs of Lemma 3.2 (which “finds” β) and the Reflection Lemma 3.3. Express $H' = \{\xi_n : n \in \omega\}$. For every $x = \langle \beta, m \rangle \in X$ let $V(x) = V_{t(x), K(x)}(x)$ be a basic neighborhood such that

$$(T_0) \quad \{\xi_j : j \leq m\} \subset t(x),$$

and thus $V(x) \subset \bigcap_{j \leq m} B_{\xi_j}^{x(\xi_j)}$. (Recall the definition of the basic open neighborhood.) By passing to

a smaller neighborhood, if needed, we will assume

$$(T_1) \quad j < m < \omega \implies t(\beta, j) \subset t(\beta, m);$$

$$(T_2) \quad V_{t(x), K(x)}(x) \text{ is a complete neighborhood.}$$

Consider $t : X \rightarrow [H]^{<\omega}$ and $K : X \rightarrow [X]^{<\omega}$ to be the functions defined above for the conditions (T_0) , (T_1) , (T_2) . Let

$$\mathfrak{c}, \langle S_\xi \rangle_{\xi < 2^{\mathfrak{c}}}, \langle \langle B_\xi^0, B_\xi^1 \rangle \rangle_{\xi \in H}, H, t, K, \langle \xi_m \rangle_{m \in \omega} \in M \in N \prec H((2^{2^{\mathfrak{c}}})^+)$$

(where M, N are countable elementary submodels and $H((2^{2^{\mathfrak{c}}})^+)$ is the collection of all sets having transitive closure of cardinality $\leq 2^{2^{\mathfrak{c}}}$).

Let $A = N \cap X (= (N \cap \mathfrak{c}) \times \omega)$ and $\mathcal{R} = \{S_\xi \upharpoonright A : \xi \in M \cap H\}$. The next lemma will give a function u to complete a special control triple $\langle A, \mathcal{R}, u \rangle$.

Lemma 3.2. *There exists a countable function u with $\text{dom}(u) \subset A$ satisfying (C_1) and (C_2) in the definition of a control triple such that whenever $v : X \rightarrow [H \setminus M]^{<\omega}$ is an infinite partial function, $v \in N$ and $x \neq x'$ in $\text{dom}(v)$ implies $v(x) \cap v(x') = \emptyset$, then there are infinitely many $x \in \text{dom}(v) \cap \text{dom}(u)$ such that*

$$u(x) = \{S_\xi \upharpoonright A : \xi \in v(x)\}.$$

Proof. Let $\langle v_j \rangle_{j \in \omega}$ enumerate all functions $v \in N$, as in the lemma, each listed infinitely many times. By induction on j pick distinct $\{x_j : j \in \omega\} \subset N \cap X$ such that $j < m < \omega$ implies $v_j(x_j) \cap v_m(x_m) = \emptyset$. Define

$$u : \{x_i : i \in \omega\} \rightarrow [S_A \setminus \mathcal{R}]^{<\omega} : u(x_i) = \{S_\xi \upharpoonright A : \xi \in v_i(x_i)\}.$$

For (C_1) to hold we need $u(x_j) \cap \mathcal{R} = \emptyset$ for all $j \in \omega$.

Assume indirectly that there exists $\xi \in v(x_j)$ and $\eta \in M \cap H$ such that $S_\xi \upharpoonright A = S_\eta \upharpoonright A$. Now, $\eta \in N$ since $\eta \in M$. To see $\xi \in N$ note that $v_j, x_j \in N$ implies $v_j(x_j) \in N$. Since $v_j(x_j)$ is finite, $\xi \in v_j(x_j) \subset N$. Since $\xi, \eta \in N$ and $S_\xi \upharpoonright A = S_\eta \upharpoonright A$ it follows that $S_\xi = S_\eta$. Since $\xi, \eta \in H$ it follows from minimality of Case 1 that $\xi = \eta$. Then $\xi \in v_j(x_j) \cap (M \cap H) = \emptyset$, a contradiction.

For (C_2) , suppose indirectly that there exists $x_i \neq x_j$ in $\text{dom}(u)$ with some $S_\sigma \upharpoonright A \in u(x_i) \cap u(x_j)$. There would be $\xi \in v(x_i)$ and $\eta \in v(x_j)$ such that $S_\xi \upharpoonright A = S_\sigma \upharpoonright A = S_\eta \upharpoonright A$. As in the previous paragraph, $\xi, \eta \in H$ and $S_\xi \upharpoonright A = S_\eta \upharpoonright A$ gives $S_\xi = S_\eta$ and $\xi = \eta$. This contradicts $v(x_i) \cap v(x_j) = \emptyset$. \square

Now, fix a u as in Lemma 3.2, and fix $\beta < \mathfrak{c}$ with $\beta \notin N$ and $\langle A, \mathcal{R}, u \rangle = \langle A_\beta, \mathcal{R}_\beta, u_\beta \rangle$. Notice that $\pi_1(A) \subset \beta$ since $A_\beta \subset \beta \times \omega$.

Reflection Lemma 3.3. *Let $k \in \omega$. Then there exists $x = \langle \alpha, k \rangle \in \text{dom}(u)$ such that*

- (R₁) $t(x) \cap M = t(\beta, k) \cap M$;
- (R₂) $\forall \xi \in t(\beta, k) \cap M, \langle \beta, k \rangle(\xi) = x(\xi)$;
- (R₃) $u(x) = \{S_\xi \upharpoonright A : \xi \in t(x) \setminus M\}$.

Proof. Let $r = t(\beta, k) \cap M \in M$, $\text{dom}(f) = r$, $f(\xi) = \langle \beta, k \rangle(\xi)$. Let $\phi(\alpha)$ be the statement:

“ $t(\alpha, k) \supset r$ and, $\forall \xi \in r, \langle \alpha, k \rangle(\xi) = f(\xi)$.”

All parameters of $\phi(\alpha)$ are in M and $\phi(\beta)$ is true. Let E be maximal so that $\phi(E)$ holds and such that $t(\alpha, k) \setminus r, \alpha \in E$, are pairwise disjoint. Since all parameters of $\phi(\alpha)$ are from M we may assume $E \in M$. Suppose indirectly that E is countable. Then $E \subset M$. If $E' = E \cup \{\beta\}$ then $E \subsetneq E'$. Now, $\phi(E')$ holds because $\phi(\beta)$ clearly holds and, for all $\alpha \in E$, $(t(\alpha, k) \setminus r) \cap (t(\beta, k) \setminus r) \subset M \cap (t(\beta, k) \setminus M) = \emptyset$. This contradicts the maximality of E so E , in fact, must be uncountable.

Let $v(x) = t(\alpha, k) \setminus r$. The argument above shows there are uncountably many $\alpha \in \mathfrak{c}$ with $\phi(\alpha)$ true such that the sets $v(x) = t(\alpha, k) \setminus r$ are pairwise disjoint. Hence, there exist uncountably many with $v(x) \cap M = \emptyset$.

Let $\psi(v)$ stand for:

“ v is an infinite function, $\text{dom}(v) \subset \mathfrak{c} \times \{k\}$, $\forall \langle \alpha, k \rangle \in \text{dom}(v), \phi(\alpha)$ is true and $v(\alpha, k) = t(\alpha, k) \setminus M$ and $x \neq x'$ in $\text{dom}(v)$ implies $v(x) \cap v(x') = \emptyset$.”

All parameters of $\psi(v)$ are in N so there exists such $v \in N$. By Lemma 3.2, there exists $x \in \text{dom}(v) \cap \text{dom}(u)$ satisfying (R_3) . The argument above shows that (R_1) and (R_2) are true. \square

Conclusion of the proof of the Main Lemma. Continue with the $\beta < \mathfrak{c}$ as given after the proof of 3.2. Aiming for a contradiction to the Main Lemma let $\theta \in H' \cap M = H'$ be the smallest counterexample for which β is not θ -homogeneous. Notice that earlier condition (T_1) insures that it is possible to pick large enough $k \in \omega$ such that for $y[k] = \{\langle \beta, j \rangle : j \leq k\}$,

$$(1_k) \quad y[k] \cap B_\theta^0 \neq \emptyset \text{ and } y[k] \cap B_\theta^1 \neq \emptyset,$$

and

$$(2_k) \quad \theta \in t(\beta, k).$$

Let $x = \langle \alpha, k \rangle$ be as in the Reflection Lemma. Notice that $x \in N$ implies $K(x) \in N$, and $K(x)$ is finite so $K(x) \subset N$. Since $\beta \notin N$ it must be true that $C_\beta \cap K(x) \subset C_\beta \cap N = \emptyset$. So, $y[k] \cap K(x) = \emptyset$. This is needed to help with the following claim.

Claim. $y[k] \subset V_{t(x) \cap \theta, K(x)}(x)$.

Proof of claim. Let $\xi \in t(x) \cap \theta$ and suppose for all $\eta \in t(x) \cap \xi$ we have

$$(I_\eta) \quad y[k] \subset B_\eta^{x(\eta)}.$$

We'll show

$$(I_\xi) \quad y[k] \subset B_\xi^{x(\xi)}.$$

Certainly, by completeness and the fact that $y[k] \cap K(x) = \emptyset$,

$$(3_k) \quad y[k] \subset V_{t(x) \cap \xi, K(x)} \subset U_\xi^{x(\xi)}.$$

Case (a) $\xi \in M \cap t(x)$. By minimality of θ it follows that there exists $i \in \{0, 1\}$ such that $C_\beta \subset B_\xi^i$. By (R_1) , $\xi \in t(\beta, k) \cap M$. By (R_2) , this $i = \langle \beta, k \rangle(\xi) = x(\xi)$. So $y[k] \subset C_\beta \subset B_\xi^{x(\xi)}$.

Case (b) $\xi \in t(x) \setminus M$. Then by Subcase 1.2 and (3_k) , $\langle \beta, j \rangle(\xi) = x(\xi)$ for every $j \leq k$. That is, $y[k] \subset B_\xi^{x(\xi)}$.

Now, (I_ξ) true for all $\xi \in t(x) \cap \theta$ implies that the claim is true.

From condition (R_1) , of the Reflection Lemma, $\theta \in t(x)$. Now, by the Claim above and the fact that $V_{t(x), K(x)}(x)$ is a complete neighborhood we see that $y[k] \subset U_\theta^{x(\theta)}$. Hence by Subcase 1.1, there exists $i \in \{0, 1\}$ such that $y[k] \subset B_\theta^{x(\theta)}$ which contradicts (1_k) . That concludes the proof of the Main Lemma. \square

4. X is not countably paracompact.

To finish the proof that X is not countably paracompact let us indirectly take closed sets $Z_m \subset \mathbb{W}_m$ such that $\cup_{m \in \omega} Z_m = X$. For every $m \in \omega$, let ξ_m be the unique element of H such that $\langle U_{\xi_m}^0, U_{\xi_m}^1 \rangle = \langle \mathbb{W}_m, X \setminus Z_m \rangle$ (uniqueness by Case 1).

Proposition 4.1. *For every $\beta \in \mathfrak{c}$ there exists $m \in \omega$ such that β is not ξ_m -homogeneous.*

Proof. Since $\cup_{n \in \omega} Z_n = X$ we can pick m with $Z_m \cap C_\beta \neq \emptyset$. For such m , $C_\beta \not\subset B_{\xi_m}^1 \subset \mathbb{W}_m$ and $C_\beta \not\subset B_{\xi_m}^1 \subset X \setminus Z_m$. \square

Using $H' = \{\xi_n : n \in \omega\}$, the previous proposition contradicts the Main Lemma and concludes our argument that X is not countably paracompact.

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Zoltán Balogh Memorial Topology Conference Abstracts

Metrizably Fibered GO-spaces

Harold R Bennett, *Texas Tech University*

We characterize GO-spaces that are metrizable fibered in terms of certain quotient spaces and in sequences of open covers.

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Forcing hereditarily separable countably compact group topologies

Dikran Dikranjan, *Department of Mathematics, Udine University, 33100 Udine, Italy*

Using forcing we produce a model of $ZFC + CH$ (with 2^c arbitrarily large) and, in this model, we obtain a characterization of the Abelian groups G (necessarily of size at most 2^c) which admit:

- (i) a hereditarily separable group topology,
- (ii) a group topology making G into an S -space,
- (iii) a hereditarily separable group topology that is either precompact, or pseudocompact, or countably compact (and which can be made to contain no non-trivial convergent sequences),
- (iv) a hereditarily separable connected and locally connected group topology that is either pseudocompact, or countably compact (and which can be made to contain no non-trivial convergent sequences),
- (v) a group topology making G into an S -space that is either precompact, or pseudocompact, or countably compact (and which also can be made without non-trivial convergent sequences, if necessary).

As a by-product, we completely describe the algebraic structure of the Abelian groups of size at most 2^c which admit, at least consistently, a countably compact group topology (possibly without non-trivial convergent sequences).

We obtain also a complete solution to a 1980 problem of van Douwen about the cofinality of the size of countably compact Abelian groups (a counter-example to van Douwen's conjecture was obtained recently by Tomita).

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Radially Compact spaces

Alan Dow, *University of North Carolina at Charlotte*

We introduce a new class of spaces which we show to be compact and pseudoradial. We also show that any finite product of these spaces remain in the class. We ask if this is an approach to establishing that a finite product of compact pseudoradial spaces is again pseudoradial.

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Proper forcing revisited

Todd Eisworth, *University of Northern Iowa*

One of the mysteries of iterated forcing theory is that there seems to be no good generalization of properness to the context of iterations with uncountable support. There are ZFC theorems (mostly due to Shelah) that show many natural conjectures are false. We will present a version of properness that does work for forcing with uncountable supports, and give an example of how it can be used.

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Coarser Connected Topologies

William Fleissner, *University of Kansas*

Let P be a separation property from this list: Hausdorff, Urysohn, regular, normal, (κ) -collectionwise normal, (κ) -collectionwise Hausdorff, metric. Let (X, τ) be a property P space with a strongly separated, closed discrete set C , $|C| = d(X) \geq \mathfrak{c}$. Then there is a coarser topology σ such that (X, σ) is a connected property P space.

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The mathematics of Zoltan T. Balogh

Gary Gruenhage, *Auburn University*

I will give an overview of Zoli's work, including a list of what I consider his six "greatest hits".

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Calibers and Tightness

István Juhász, *Mathematical Institute, Hungarian Academy of Sciences*

We strengthen results of Shapirovskii and of Archangelskii, respectively by proving that

- (1) if a T_3 space X is the union of ω_1 compact subspaces of countable tightness and ω_1 is a caliber of X then X is separable;
- (2) if a T_3 space X has no uncountable free sequences and ω_1 is a caliber of X then X has a dense subset of size at most continuum.

Under CH, (2) implies that any Lindelöf T_3 space of countable tightness is separable if it has ω_1 as a caliber. However, generic left separated spaces yield (consistent) examples which show that the bound continuum given by (2) [or Archangelskii's original result] for the density is sharp. We also present a (consistent) counterexample to a problem of Okunev and Tkachuk, namely a compact T_2 space such that ω_1 is a caliber of every dense subspace but it has uncountable d -tightness.

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Hereditarily normal compactifications of metrizable spaces

Heikki Junnila, *University of Helsinki*

Hereditarily normal compactifications of metrizable spaces Abstract: We show that the non-separable metrizable hedgehog-space $J(\omega_1)$ cannot be embedded in any hereditarily normal \aleph_1 -compact space. In particular, $J(\omega_1)$ has no monotonically normal compactification; this answers a question made by P. Gartside. There also exist strongly paracompact metrizable spaces without monotonically normal compactifications; this follows from the result that a metrizable space X is locally separable if the product space $X \times \mathbb{I}$ has a hereditarily normal \aleph_1 -compactification.

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On semi- θ -generalized closed sets

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The aim of this paper is to introduce and study the concept of semi- θ -generalized closed sets in topological spaces via the semi- θ -closure operator which was introduced by G. Di Maio and T. Noiri [2]. Semi-R1, $T_{3/4}$ and gs-regular spaces are characterized. The concepts of $Ts\theta g$ -spaces and semi- θ -generalized continuous functions are introduced.

A subset A of a space X is called a *Semi- θ -generalized closed set* (denoted by $s\theta g$ -closed) if $sCl(A)$ is a subset of G whenever A is a subset of G and G is open in X . The class of all $s\theta g$ -closed subsets of X is denoted by $S\theta GC(X)$. The complement of each $s\theta g$ -closed subsets of X is called $s\theta g$ -open and the class of all $s\theta g$ -open subsets of X is denoted by $S\theta GO(X)$. A function $f : X \rightarrow Y$ is called semi- θ -generalized continuous ($s\theta g$ -continuous) if the inverse image of each closed subset of Y is semi- θ -generalized closed. Equivalently, f is semi- θ -generalized continuous if the inverse image of each open subset of Y is semi- θ -generalized open.

The following lemmas show that the class of $S\theta GC(X)$ is properly placed between the class of θ -generalized closed sets and generalized semi-closed sets and the class of semi- θ -generalized continuous functions is properly placed between the class of gs-continuous functions and θ -g-continuous functions

- (i) Each semi- θ -closed set in X is $s\theta g$ -closed.
- (ii) Each θ -generalized closed subset of X is semi- θ -generalized closed.
- (iii) Each semi- θ -generalized continuous function is gs-continuous.
- (iv) Each θ -g-continuous function is semi- θ -generalized continuous

Also we give some characterizations of semi-R1 and $T_{3/4}$ -spaces.

- (i) A topological space X is semi-R1 if and only if $sCl\{x\} = sCl\theta\{x\}$ for each x in X .
- (ii) A topological space X is $T_{3/4}$ -space if and only if each semi- θ -generalized closed set is semi- θ -closed.
- (iii) A topological space X is $T_{3/4}$ -space if and only if for each semi- θ -generalized closed F in X and each x in F , there exist disjoint semi-open sets U and V containing x and F respectively.

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κ -Fréchet-Urysohn Spaces

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(Arhangel'skii, 1999) A Hausdorff topological space X is called κ -Fréchet-Urysohn if for every open subset A of X and every $x \in \bar{A}$ there exists a sequence of points of A converging to x . We will discuss the properties of κ -Fréchet-Urysohn spaces and several results including: The regular topological space X is κ -Fréchet-Urysohn if and only if X is a κ -pseudo-open image of a metric space.

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Trees and lines

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Given linear orderings for its nodes, a tree (T, \leq) gives rise to lines (= linearly ordered sets) in two very different ways, namely by branch space constructions and by lexicographic orderings of the tree itself. Without further restrictions, any line is order isomorphic to the branch space of some tree, and also to some lexicographically ordered tree. However, with reasonable restrictions on the tree, the situation is quite different.

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Fat-Psi constructions

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We survey some old and some new results obtained by means of fat-Psi constructions.

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Balogh's 'lost' theorems on paracompactness in locally compact spaces

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In the 1988 volume of AMS Abstracts, Zoltán Balogh announced two striking applications of large cardinals to the theme of when a locally compact space is paracompact. Both applications involved the concept of "strongly ω_1 -cwH." This is the property that every closed discrete subspace of cardinality $\leq \omega_1$ expands to a discrete collection of open sets. If "discrete" is weakened to "disjoint," we get the more familiar concept of " ω_1 -cwH."

The first announced application used Fleissner's Axiom R; the second used Martin's Maximum. Application 1: Every locally compact, $\delta\theta$ -refinable, strongly ω_1 -cwH space is paracompact. Application 2: Every locally compact, hereditarily strongly ω_1 -cwH space is either paracompact or contains a perfect preimage of ω_1 .

The second application has finally appeared in print. It is in a paper by Zoltán titled “Locally nice spaces and Axiom R,” which appeared a few days before the conference: *Topology and its Applications* 125 (2002) 335–341. The fragment of Martin’s Maximum actually used is $MA(\omega_1) +$ Axiom R. It has several other striking results which will be recounted in this talk along with some strengthenings due to Alan Dow and the speaker.

The first application is still shrouded in mystery. I do not even know whether it is valid. With “strongly cwH” in place of “strongly ω_1 -cwH,” we have a ZFC theorem, also due to Zoli, and appearing already in “Paracompactness in locally Lindelof spaces”, *Canad. J. Math.* 38(1986), 719-727. I will show how Application 1 is valid for countably tight spaces.

A ZFC example of a locally compact, metacompact, screenable, hereditarily cwH non-normal space with a countable T_0 -separating open cover will also be presented, showing just how essential the word “strongly” is in Zoli’s theorems.

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Base-cover and base-family paracompactness

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We present results from the author’s dissertation (Auburn University, August 2002). A space is base-cover paracompact if it has an open base every subcover of which has a locally finite subcover. A space is base-family paracompact if it has an open base every subfamily of which has a subfamily with the same union and which is locally finite at each point from its union. Several open questions are posed too.

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An α -normal non- β -normal Moore space

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The notions of α -normal and β -normal spaces were introduced by A. Arhangel’skii and L. Ludwig. A. Arhangel’skii and L. Ludwig showed that if $2^\omega < 2^{\omega_1}$ then separable, α -normal Moore spaces are metrizable. It is shown that if there exists a separable, α -normal, non-metrizable Moore space then there exists a Q-set. Also, M. Wage’s example of a collectionwise Hausdorff, non-metrizable Moore space is shown to be collectionwise α -normal but not β -normal.

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Computational models of topological spaces

Mike Reed, *Oxford University Computing Laboratory*

Two major questions in domain theory are:

Can each complete Moore space (complete metrizable space) be realized as the maximal elements of a Scott domain?

The author shows that the answer is negative for complete Moore spaces and provides a positive (partial) result for metrizable spaces.

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A joint paperMary Ellen Rudin, *University of Wisconsin*

Zoli and I were attracted by similar ideas and often even by the same problem. I gloried in his solutions, but once we wrote a joint paper!

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FUF and boundedly FUF spacesPaul Szeptycki, *York University*

The class of FUF-spaces (recently introduced by Reznichenko and Sipacheva) is a subclass of Frechet-Urysohn spaces. A space is FUF if for every point x and every family F of finite subsets of X , if F forms a local π -net at x , then F has a subset that converges to x .

We investigate the relationship of FUF to other classes of Frechet-Urysohn spaces including the α_i -spaces of Arhangel'skii.

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Optimal constant weight binary codesPrem Sharma, *Butler University*

Let X be an n -set and $\binom{X}{w}$ the collection of all w -subsets of X equipped with the Hamming distance. A subset of $\binom{X}{w}$ is called a constant weight code and its members are codewords. The distance in a given code means the minimum distance between a pair of distinct codewords. The problem of determining the optimal size of a constant weight code having a preassigned distance in it is discussed.

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Riffs on Zoli’s “Locally nice spaces under Martin’s axiom”

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In the cited paper, Balogh extended Szentmiklóssy’s “no compact S -spaces” theorem to get that, under $MA + \sim CH$,

(B) *Locally countable subspaces of size $< \mathfrak{c}$ are σ -discrete in compact spaces of countable tightness.*

Balogh used this to obtain a large number of interesting consequences. In joint work with Paul Larson, we prove that (B) is consistent with:

(C) *Every normal first countable space is collectionwise Hausdorff.*

This enables us to obtain a variety of consistency results along the same lines as Balogh’s but more powerful. Our results depend on work of Todorćević which currently requires supercompact cardinals, but that requirement can probably be eliminated. Assuming then the consistency of a supercompact cardinal, we obtain the consistency of

- (i) *locally compact, locally hereditarily Lindelöf, hereditarily normal spaces are paracompact if and only if they do not include a perfect pre-image of ω_1 ,*
- (ii) *locally compact perfectly normal spaces are paracompact,*
- (iii) *locally compact hereditarily normal spaces with G_δ -diagonals are metrizable,*
- (iv) *locally compact perfectly normal spaces of size \aleph_1 are metrizable,*
- (v) *locally compact spaces with hereditarily normal squares are metrizable.*

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The Urysohn Property and two step iteration of almost disjoint families

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Let \mathcal{A}_0 be an infinite maximal almost disjoint family of infinite subsets of the natural numbers ω , and $\psi(\mathcal{A}_0) = \omega \cup \mathcal{A}_0$ with the topology in which for every natural number $n \in \omega$, the singletons $\{n\}$ are isolated for $n \in \omega$ and each $A \in \mathcal{A}_0$ has a local base consisting of sets of the form $\{A\} \cup A \setminus F$ where F is a finite subset of ω . This is a well-known space in topology. We consider a collectionwise type separation property on $\psi(\mathcal{A}_0)$ which can be equivalently stated as the Urysohn separation property on an extension of $\psi(\mathcal{A}_0)$. The extension is defined from $\psi(\mathcal{A}_0)$ by taking \mathcal{A}_1 , an infinite, maximal almost disjoint family of countable subsets of the non-isolated points of $\psi(\mathcal{A}_0)$, and defining neighborhood of an elements $X \in \mathcal{A}_1$ by $\{X\} \cup (X \setminus G) \cup (\cup\{A \setminus F(A) : A \in X \setminus G\})$ where G is finite, and $F(A)$ is finite for all $A \in X \setminus G$. The resulting space is denoted $\psi(\mathcal{A}_0, \mathcal{A}_1)$. It is consistent that there exists $\mathcal{A}_0, \mathcal{A}_1$ such that $\psi(\mathcal{A}_0, \mathcal{A}_1)$ is Urysohn. We will prove in ZFC that there exists a maximal \mathcal{A}_0 such that for every maximal \mathcal{A}_1 , $\psi(\mathcal{A}_0, \mathcal{A}_1)$ is not Urysohn.

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Monotone Extensions

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We characterize the closed subsets H of a monotonically normal space X that allow an extender $e : C(H) \rightarrow C(X)$ such that

- (1) If f is a continuous function from H into the reals, ef is a continuous extension of f .
- (2) If f and g are in $C(H)$ with $f(x) \leq g(x)$ for all $x \in H$, then $f(x) \leq g(x)$ for all $x \in X$.

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Zoltán Balogh Memorial Topology Conference Contributed Problems

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Title: **A bunch of Problems on D -spaces**

We consider below only T_1 -spaces. The notion of a D -space was introduced by E. van Douwen in [5]. A *neighbourhood assignment* on a topological space X is a mapping ϕ of X into the topology \mathcal{T} of X such that $x \in \phi(x)$, for each $x \in X$. A space X is a D -space if, for every neighbourhood assignment ϕ on X , there exists a closed discrete subset A of X such that the family $\phi(A)$ covers X . A principal property of D -spaces is that the extent of any D -space coincides with the Lindelöf number. In particular, every countably compact D -space is compact, and every D -space with the countable extent is Lindelöf.

Problem (1). (E. van Douwen [5]) *Is every regular Lindelöf space a D -space?*

It is even unknown if every hereditarily Lindelöf regular T_1 -space is a D -space.

Problem (2). (E. van Douwen [5]) *Is every regular (Tychonoff) subparacompact space a D -space?*

Recall that a space X is *subparacompact* if every open covering of X can be refined by a σ -discrete closed covering [3]. All metrizable spaces, and, more generally, all Moore spaces are D -spaces [2]. A much more general result was recently obtained by R.Z. Buzyakova: every strong Σ -space is a D -space [4]. Hence, all Tychonoff spaces with a countable network, all σ -spaces, all paracompact p -spaces, and all Lindelöf Σ -spaces are D -spaces. On the other hand, there exists a locally compact σ -metrizable Tychonoff space with the diagonal G_δ which is not a D -space [6].

A space X is an *aD-space* [1] if for each closed subset F of X and each open covering γ of X there exist a closed discrete subset A of F and a mapping ϕ of A into γ such that $a \in \phi(a)$, for each $a \in A$, and the family $\phi(A) = \{\phi(a) : a \in A\}$ covers F . Every subparacompact space is an *aD-space* [2]. Notice that every *aD-space* of the countable extent is Lindelöf.

Problem (3). [1] *Is every aD-space X a D -space? What if X is, in addition, regular (Tychonoff)?*

The positive answer to Problem 3 would also solve Problems 1 and 2.

Several open problems on D -spaces are related to the following general question: how complex can be a space which is the union of two “nice” subspaces? The Alexandroff compactification of any uncountable discrete space ω_1 is the union of two metrizable (in fact, discrete) subspaces, while it is not first countable and, therefore, not metrizable. Another example of a non-metrizable compactum, which is the union of two metrizable subspaces, is the double circumference of Alexandroff and Urysohn. This space is first countable and, hence, Fréchet-Urysohn.

If a regular space X is the union of finitely many metrizable subspaces, then X is a D -space [1]. If a regular space X is the union of a countable family γ of dense metrizable subspaces, then X is a D -space [1]. Similarly, if a space X is the union of a countable family of open metrizable subspaces, then X is also a D -space [1].

Problem (4). [1] *Suppose that $X = Y \cup Z$, where Y and Z are D -spaces. Is then X a D -space? Is then X an aD-space? What if X is, in addition, assumed to be Tychonoff?*

Problem (5). [1] *Suppose that $X = Y \cup Z$, where Y and Z are D -spaces, and X is countably compact. Must X be compact? What if X is, in addition, assumed to be Tychonoff?*

We do not even know whether every countably compact space X which is the union of a countable family of D -spaces must be compact.

Problem (6). *Suppose that X is the union of a countable family of open D -subspaces. Must X be a D -space?*

In connection with Problems 4 and 5, notice the following simple result [1]: If $X = Y \cup Z$, where Y and Z are aD -spaces (D -spaces) and Y is closed in X , then X is also an aD -space (a D -space, respectively).

Problem (7). *Suppose that X is a Lindelöf D -space and Y is a continuous image of X . Must Y be a D -space? What if Y is, in addition, assumed to be regular (Tychonoff)?*

Here is an interesting special version of Problems 4 and 7.

Problem (8). *Suppose that $X = Y \cup Z$, where Y and Z are Lindelöf D -spaces. Is then X a D -space? What if X is, in addition, assumed to be regular?*

Below $C_p(\omega_1)$ is the space of continuous real-valued functions, in the pointwise topology, on the space ω_1 of countable ordinals (with the order topology).

Problem (9). *Is $C_p(\omega_1)$ a D -space?*

Note, that $C_p(\omega_1)$ is Lindelöf.

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Title: The Namioka properties

A completely regular space B has the Namioka property if for every separately continuous $f : B \times K \rightarrow [0, 1]$, with K compact, f is jointly continuous at each point of a set $A \times K$, where A is dense in B .

Problem (A). *Let B be a completely regular space such that for every separately continuous $f : B \times K \rightarrow [0, 1]$, with K compact, and each closed F is $B \times K$ projecting irreducibly onto B , the set of points of continuity of the restriction $f|_F : F \rightarrow [0, 1]$ is dense in F . Does B have the Namioka property?*

The Namioka property implies the property in Problem A, and this property yields that B is a Baire space, cf. [BP1]. Let us also notice that there is a Baire space B (even a Choquet space) and a separately continuous $f : B \times \beta B \rightarrow [0, 1]$ such that the restriction of f to the diagonal $\{(b, b); b \in B\}$, has no continuity points [BP1].

Dually, a compact space K is a Namioka space, if for any separately continuous $f : B \times K \rightarrow [0, 1]$, with B Baire, f is continuous at each point of some set $A \times K$, where A is dense in B . We shall denote by $(C(K), \text{weak})$, $(C(K)^*, \text{weak}^*)$ the space of real-valued continuous functions on K , and its dual space (identified with the space of Radon measures on K), endowed with the topology of pointwise convergence on $C(K)^*$, or $C(K)$, respectively.

Problem (B). *Let K be a compact Namioka space. Is the duality mapping $(f, \mu) \rightarrow \int f d(\mu)$ Borel measurable on $(C(K), \text{weak}) \times (C(K)^*, \text{weak}^*)$?*

Let us notice that the duality mapping is not Borel for any infinite compact F -space K , cf. [BP2]. For relevant information on this topic we refer also to a paper by Maxim Burke, *Borel measurability of separately continuous functions* [Bu]. For additional information on the Namioka properties see the article by Mercourakis and Negrepontis [MN].

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Title: **Products of Baire spaces**

A space X is hereditarily Baire if all its closed subspaces are Baire. For metrizable X , this means that X contains no closed copy of the rationals.

Problem. *If X, Y are metrizable spaces, X is Baire, and Y is hereditarily Baire, is then the product $X \times Y$ a Baire space?*

There are metrizable Baire spaces X, Y whose product is not Baire, cf. Fleissner and Kunen [2]. On the other hand, arbitrary products of metrizable hereditarily Baire spaces are Baire, cf. [1]. Let us also notice that if a metrizable hereditarily Baire space is split into countably many sets, then one of them must contain a relatively open nonempty subspace whose product with any metrizable hereditarily Baire space is Baire, cf. [1].

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Title: **Some Cardinality Questions Concerning Countably Compact Spaces**

Dedicated to the Memory of Zoltan Balogh

Remark. The reader will lose little by interpreting the following Questions, and the results cited from the literature, within the class of Tychonoff spaces. In fact, however, several of those theorems are meaningful and valid when the expression “countably compact” is taken in the widest possible sense, that is, with no separation properties assumed; others must be construed within the class of Tychonoff spaces.

It was shown by SCARBOROUGH and A. H. STONE [8](5.6) that a product $\prod_{i \in I} X_i$ of nonempty spaces is countably compact if and only if each subproduct $\prod_{i \in J} X_i$ with $J \subseteq I$ and $|J| \leq 2^{2^c}$ is countably compact. Later, using the theory of p -limits and p -compactness ($p \in \omega^* := \beta(\omega) \setminus \omega$) introduced by BERNSTEIN [1], GINSBURG and SAKS [5](2.6) showed for a space X that every power X^κ is countably compact if and only if X^{2^c} is countably compact. Prior to the appearance of [1],

FROLÍK [4] had introduced a notion equivalent to the p -limit concept and had used it to show in ZFC that ω^* is not homogeneous, and also to produce a sequence of spaces X_n ($n < \omega$) such that each $\prod_{i \in F} X_i$ with $F \in [I]^{<\omega}$ is countably compact, while $\prod_{n < \omega} X_n$ is not countably compact. With the results of [8], [5] and [4] before them, SAKS [7] and COMFORT [2] then proved:

Theorem 1. A product $\prod_{i \in I} X_i$ of nonempty spaces is countably compact if and only if each subproduct $\prod_{i \in J} X_i$ with $J \subseteq I$, $|J| \leq 2^c$, is countably compact.

Theorem 1 makes natural the following question, which was asked already in [2] and subsequently elsewhere.

Question (1). *In Theorem 1, is the cardinal number 2^c best possible?*

Question 1 has some close relatives, as follows.

Question (2). *Is there a set $\{X_i : i \in I\}$ of spaces, with $|I| = 2^c$, such that $\prod_{i \in I} X_i$ is not countably compact but $\prod_{i \in J} X_i$ is countably compact for each proper $J \subseteq I$?*

Question (3). *Is there a space X such that X^{2^c} is not countably compact, but X^κ is countably compact for each $\kappa < 2^c$?*

Much is known about these questions. For example, SAKS [7] answered Question 2 affirmatively, if MA is assumed. YANG [11] showed that Question 3 has an affirmative answer if and only if every set $S \subseteq \omega^*$ with $|S| < 2^c$ satisfies $\omega^* \neq A(S)$. Here $A(S)$ is the set of ultrafilters which sit “above” some element of S in the Rudin-Keisler order; that is, $A(S)$ is the set of $q \in \omega^*$ such that for some $f : \omega \rightarrow \omega$ the Stone extension $\bar{f} : \beta(\omega) \rightarrow \beta(\omega)$ satisfies $\bar{f}(q) \in S$. Accordingly, all three questions have consistent positive solutions. What is desirable in each case is an absolute proof (in ZFC), or a consistent proof of the negation.

The questions can be formulated in the context of topological groups. Here is a relevant ZFC result.

Theorem 2. In order that every product of countably compact topological groups be countably compact, it is necessary and sufficient that there exists $p \in \omega^*$ such that each countably compact topological group is p -compact.

[The proof of necessity follows the argument of the proof of the theorem cited above from [2] and [7], while sufficiency is a consequence of BERNSTEIN’s theorem [1] that (for fixed $p \in \omega^*$) the product of any set of p -compact spaces is p -compact (and hence countably compact).]

Assuming CH, VAN DOUWEN [3] showed the existence of two countably compact topological groups whose product is not countably compact. K. P. HART and VAN MILL [6] then showed, using $\text{MA}_{\text{countable}}$, that there is a countably compact group G such that $G \times G$ is not countably compact. These results were improved by TOMITA [9], [10], who showed *inter alia*, always assuming $\text{MA}_{\text{countable}}$, that for $0 < k < \omega$ there is a sequence G_n ($n < \omega$) of countably compact topological groups such that each $\prod_{n \in F} G_n$ is countably compact when $F \in [\omega]^{\leq k}$, but $\prod_{n \in F} G_n$ is not countably compact when $F \in [\omega]^{\geq k+1}$. Those results suggest these questions.

Question (4). *Does there exist in ZFC a set G_i ($i \in I$) of countably compact topological groups such that $\prod_{i \in I} G_i$ is not countably compact? With $|I| < 2^c$? With $|I| = 2$? With $\prod_{i \in J} G_i$ countably compact for all $J \subseteq I$ with $|J| < |I|$? With $\prod_{i \in J} G_i$ countably compact for all proper $J \subseteq I$?*

Question (5). *Are the (equivalent) conditions given in Theorem 2 consistent with ZFC?*

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Title: Countably compact groups without nontrivial convergent sequences

Being a dyadic space, every infinite compact topological group has nontrivial convergent sequences. This inspired interest in compact-like group topologies without non-trivial convergent sequences (see, for example, [8, 5, 4, 6, 7, 9, 1, 3, 11, 2]).

Denote by \mathcal{C} the class of Abelian groups that admit a countably compact group topology and by \mathcal{P} the class of Abelian groups that admit a pseudocompact group topology. In [2] the following statement has been shown to be consistent with ZFC: If G is a group from the class \mathcal{C} (or the class \mathcal{P}) and $|G| \leq 2^{\mathfrak{c}}$, then G admits a hereditarily separable countably compact (respectively, pseudocompact) group topology without non-trivial convergent sequences. This naturally brings us to the following

Question 4.2. Does every group G from the class \mathcal{C} (or \mathcal{P}) has a countably compact (respectively, pseudocompact) group topology without non-trivial convergent sequences?

The next question, going in the opposite direction, may be considered as a “countably compact heir” of the fact that compact groups have non-trivial convergent sequences that still has a chance of a positive answer in ZFC.

Question 4.3. Let G be an infinite group from class \mathcal{C} . Does G has a countably compact group topology that contains a non-trivial convergent sequence?

Since all known examples of countably compact groups without non-trivial convergent sequences seem to be Abelian, it seems reasonable to ask what happens for *essentially* non-commutative groups.

Question 4.4. Let G be an infinite countably compact group without open Abelian subgroups. Does G have a non-trivial convergent sequence?

Question 4.5. Let G be an infinite countably compact group with trivial center $Z(G) = \{g \in G : xg = gx \text{ for each } x \in G\}$. Does G have a non-trivial convergent sequence?

Martin’s Axiom MA yields positive answers to both Question 4.4 and Question 4.5 for groups of weight $< \mathfrak{c}$ [1]. A *consistent* negative answer to Question 4.5 was given in [2]. Furthermore, the example may be chosen hereditarily separable and locally connected (or hereditarily separable and totally disconnected).

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Title: Compact-like group topologies on Abelian groups

Halmos [11] asked which Abelian groups admit compact group topologies, and complete solution to this problem has been found by Harrison [12] and Hulanicki [13]. The counterpart of Halmos’ problem for pseudocompact groups was attacked in [3, 4, 1, 2, 5] and complete solutions have been found for major classes of groups. Nevertheless, the general case still remains open:

Problem 4.6. Describe an algebraic structure of pseudocompact Abelian groups.

The question of which Abelian groups admit a countably compact group topology happens to be much more complicated. Let G be an Abelian group. As usual $r(G)$ denotes the *free rank* of G . For every natural number $n \geq 1$ define $G[n] = \{g \in G : ng = 0\}$ and $nG = \{ng : g \in G\}$. It is relatively easy to see that every countably compact group G satisfies the following two conditions [3, 5, 3]:

PS: Either $r(G) \geq \mathfrak{c}$ or $G = G[n]$ for some $n \in \omega \setminus \{0\}$.

CC: For every pair of integers $n \geq 1$ and $m \geq 1$ the group $mG[n]$ is either finite or has size at least \mathfrak{c} .

Definition 4.7. (i) We denote by \mathcal{C} the class of Abelian groups that admit a countably compact group topology.

(ii) We denote by \mathcal{P} the class of Abelian groups that admit a pseudocompact group topology.

By the above result, every group G from the class \mathcal{C} satisfies both **PS** and **CC**.

After a series of scattered results [4, 8, 9, 18], a complete description of the algebraic structure of members of \mathcal{C} that have size at most \mathfrak{c} has been recently obtained, under Martin’s Axiom MA, in [7]: MA implies that an Abelian group G of size at most \mathfrak{c} belongs to \mathcal{C} if and only if it satisfies both **PS** and **CC**. (In particular, every torsion-free Abelian group of size \mathfrak{c} belongs to \mathcal{C} under MA [16].) This result has been substantially extended in [6]:

Theorem 4.8. *There exists a model of ZFC in which the following three statements hold (simultaneously):*

(1) *A group G of size at most $2^{\mathfrak{c}}$ belongs to \mathcal{C} if and only if it satisfies both **PS** and **CC**.*

(2) *An Abelian group G of size at most $2^{\mathfrak{c}}$ satisfying both **PS** and **CC** admits a hereditarily separable countably compact group topology without non-trivial convergent sequences.*

(3) *An Abelian group G admits a countably compact separable group topology if and only if $|G| \leq 2^{\mathfrak{c}}$ and G satisfies both **PS** and **CC**.*

This recent “jump” from \mathfrak{c} to $2^{\mathfrak{c}}$ is an essential step forward with respect to the previous knowledge about countably compact group topologies on groups of size bigger than \mathfrak{c} . In fact, this “prior knowledge” was essentially limited to the following two facts: It is consistent with ZFC that a free

Abelian group of size $2^{\mathfrak{c}}$ belongs to \mathcal{C} [14] and it is also consistent with ZFC that an Abelian group of size \aleph_ω (with $\mathfrak{c} < \aleph_\omega < 2^{\mathfrak{c}}$) belongs to \mathcal{C} [19].

While an algebraic description of Abelian groups admitting either a compact or a pseudocompact group topology can be carried out without any additional set-theoretic assumptions beyond ZFC, all substantial results about countably compact topologizations described above have either been obtained by means of some additional set-theoretic axioms (usually Continuum Hypothesis CH or versions of MA) or their consistency has been proved by forcing.

The lack of any ZFC results about separable countably compact topologies on Abelian groups justifies our next problem:

Problem 4.9. Describe in ZFC an algebraic structure of groups of size at most $2^{\mathfrak{c}}$ that belong to \mathcal{C} .

The following special version of this problem is also open:

Problem 4.10. Describe in ZFC an algebraic structure of separable countably compact Abelian groups.

The next two question provides a natural hypothesis for the solution of the above problems:

Question 4.11. Is it true in ZFC that an Abelian group G of size at most $2^{\mathfrak{c}}$ belongs to \mathcal{C} if and only if G satisfies both **PS** and **CC**?

Question 4.12. Is it true in ZFC that an Abelian group G admits a separable countably compact group topology if and only if $|G| \leq 2^{\mathfrak{c}}$ and G satisfies both **PS** and **CC**?

Note that Theorem 4.8 provides a strong positive *consistent* answer to the last two questions.

Recall that an Abelian group G is *divisible* provided that for every $g \in G$ and each positive integer n one can find $h \in G$ such that $nh = g$. An Abelian group is *reduced* if it does not have non-zero divisible subgroups. Every Abelian group G admits a unique representation $G = D(G) \oplus R(G)$ into the maximal divisible subgroup $D(G)$ of G (the so-called *divisible part of G*) and the reduced subgroup $R(G) \cong G/D(G)$ of G (the so-called *reduced part of G*). It is well-known that an Abelian group G admits a compact group topology if and only if both its divisible part $D(G)$ and its reduced part $R(G)$ admit a compact group topology. However, there exist groups G and H that belong to \mathcal{P} but neither $D(G)$ nor $R(H)$ belongs to \mathcal{P} [5, Theorem 8.1 (ii)]. This was “strengthened” in [3, 2] as follows: It is consistent with ZFC that there exist groups G' and H' from the class \mathcal{C} such that neither $D(G')$ nor $R(H')$ belong to \mathcal{P} . These results leave open the following

Problem 4.13. In ZFC, give an example of groups G and H from the class \mathcal{C} such that:

- (i) $D(G)$ does not belong to \mathcal{C} (or even \mathcal{P}),
- (ii) $R(H)$ does not belong to \mathcal{C} (or even \mathcal{P}).

Even the following question is also open:

Question 4.14. Let G be a group in \mathcal{C} .

- (i) Is it true that either $D(G)$ or $R(G)$ belongs to \mathcal{C} ?
- (ii) Must either $D(G)$ or $R(G)$ belong to \mathcal{P} ?

We note that item (ii) of the last question is a strengthening of Question 9.8 from [5]. Even consistent results related to the last question are currently unavailable.

Assuming MA, there exist countably compact groups G, H such that $G \times H$ is not countably compact [9]. Therefore, our next question could be viewed as a “weaker form” of productivity of countable compactness in topological groups that still has a chance for a positive answer.

Question 4.15. If G and H belong to \mathcal{C} , must then their product $G \times H$ also belong to \mathcal{C} ?

In fact, one can consider a much bolder hypothesis:

Question 4.16. Is \mathcal{C} closed under *arbitrary* products? That is, if G_i belongs to \mathcal{C} for each $i \in I$, does then $\prod_{i \in I} G_i$ belong to \mathcal{C} ?

Our next question is motivated by a well-known theorem of Ginsburg and Saks [10] in the framework of topological spaces where 2^c can be taken for τ below:

Question 4.17. Is there a cardinal τ having the following property: A product $\prod_{i \in I} G_i$ belongs to \mathcal{C} provided that $\prod_{j \in J} G_j$ belongs to \mathcal{C} whenever $J \subseteq I$ and $|J| \leq \tau$?

In particular, is \mathfrak{c} or 2^c such a cardinal?

A partial positive answer to Question 4.16 has been given in [6]: It is consistent with ZFC that, for every family $\{G_i : i \in I\}$ of groups with $2^{|I|} \leq 2^c$ such that G_i belongs to \mathcal{C} and $|G_i| \leq 2^c$ for each $i \in I$, the product $\prod_{i \in I} G_i$ also belongs to \mathcal{C} . A similar result for much smaller products and much smaller groups has been proved in [7, Theorem 5.6] under the assumption of MA. In particular, if the groups G and H in Question 4.15 are additionally assumed to be of size at most 2^c , then the positive answer to this restricted version of Question 4.15 is consistent with ZFC [6].

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Title: **Strong Extension Principle**

Strong Extension Principle (sEP): Every continuous function $f: \omega^* \rightarrow \omega^*$ can be extended to a continuous function $g: \beta\omega \rightarrow \beta\omega$.

Question: *Is Strong Extension Principle consistent with ZFC?*

I conjecture that PFA implies the Strong Extension Principle. Shelah [1] has proved that it is consistent with ZFC that all autohomeomorphisms of ω^* are trivial. This implies that every autohomeomorphism of ω^* continuously extends to a map $f: \beta\omega \rightarrow \beta\omega$. Later Shelah-Steprans and Velickovic deduced this from PFA and weaker axiom OCA+MA, respectively. I have proved that OCA+MA imply that for every continuous $f: \omega^* \rightarrow \omega^*$ there is a clopen $U \subseteq \omega^*$ such that the restriction of f to U continuously extends to $\beta\omega$ and the image of $\omega^* \setminus U$ is nowhere dense. This is the simplest instance of the weak Extension Principle, see Chapter 4 of [2]. By ([3]), sEP is equivalent to an apparently stronger statement that for every cardinal κ every continuous $f: (\omega^*)^\kappa \rightarrow \omega^*$ can be extended to a continuous function $g: (\beta\omega)^\kappa \rightarrow \beta\omega$. K.P. Hart has observed that sEP implies that every extremally disconnected continuous image of ω^* is separable.

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Title: **Elementary sets**

Let X be a space with base \mathcal{B} . Sets of the form

$$E = \bigcap \{B_i : 0 \leq i < n\} \cap \bigcup \{X \setminus B_i : n \leq i < m\}$$

where each $B_i \in \mathcal{B}$ is called elementary. If we change the base \mathcal{B} , we (probably) change the elementary sets. Call a base \mathcal{B} nice if every nonempty elementary set has nonempty interior.

Problem. *Which spaces X have nice bases?*

Clearly zero-dimensional spaces do. By the method of “distinct endpoints”, \mathbb{I} , \mathbb{I}^n , and \mathbb{I}^ω do, too.

Problem. *Does every compact metric space have a nice base?*

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Title: **Metrisability of manifolds and topological games**

For a topological space X , Matveev [2] introduced the following, successively stronger, conditions:

- A space X has *property (a)* provided that for every open cover \mathcal{U} of X and every dense subset $D \subset X$ there is a subset $F \subset D$ such that F is a closed and discrete subspace of X and $st(F, \mathcal{U}) = X$.
- A space X is *a-favourable* provided that for every open cover \mathcal{U} of X there is a winning strategy for the second player in the following topological game: at the α th step the first player chooses a dense subspace $D_\alpha \subset X$ then the second player chooses a point $x_\alpha \in D_\alpha$; the second player wins if for some α the set $F_\alpha = \{x_\beta / \beta < \alpha\}$ is closed and discrete in X and $st(F_\alpha, \mathcal{U}) = X$.

- A space X is *strongly a -favourable* provided that for every open cover \mathcal{U} of X there is a winning strategy for the first player in the following topological game: at the α th step the first player chooses a non-empty open set $O_\alpha \subset X$ then the second player chooses a point $x_\alpha \in O_\alpha$; the first player wins if for some α the set $F_\alpha = \{x_\beta / \beta < \alpha\}$ is closed and discrete in X and $st(F_\alpha, \mathcal{U}) = X$.
- A space X has *property pp* provided that every open cover \mathcal{U} of X has an open refinement \mathcal{V} consisting of non-empty sets such that for every choice function $f : \mathcal{V} \rightarrow X$ (ie $f(V) \in V$ for each $V \in \mathcal{V}$) the set $f(\mathcal{V})$ is closed and discrete in X .

Weakened versions of each property are obtained by replacing “closed and discrete” by “discrete”, and are denoted by putting a w before the a or p , with obvious implications.

In [1] it is shown that every manifold is strongly wa favourable and that every manifold with property wpp is metrisable while examples are given of a manifold which does not have property (a) and of a non-metrisable manifold which is strongly a -favourable.

Problem (1). *Is there a manifold which has property (a) but which is not a -favourable?*

Problem (2). *Is there a manifold which is a -favourable but which is not strongly a -favourable?*

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Title: **Dowker Filters**

C.H. Dowker posed the following problem in 1952:

Problem. *Does there exist (in ZFC) a set X and a filter \mathcal{F} on X such that:*

- for every partition $e : X \rightarrow 2$ of X , there is an $F : X \rightarrow \mathcal{F}$ such that if $x, y \in X$ and $e(x) \neq e(y)$, then either $x \notin F(y)$ or $y \notin F(x)$;*
- there is no function $F : X \rightarrow \mathcal{F}$ such that, for every $x, y \in X$, $x \neq y$ implies either $x \notin F(y)$ or $y \notin F(x)$?*

Call such a filter, if it exists, a *Dowker filter*. Dowker showed that there do not exist Dowker filters on any set of size $\leq \omega_1$. Zoli and I showed in 1991 that it is consistent for there to Dowker filters on ω_2 (among other cardinals). But it is still not known if there is a Dowker filter in ZFC.

The paper of Zoli and I on Dowker filters came after we read a paper of Mary Ellen Rudin in which she uses a certain technique for answering a topological problem related to Dowker's. This technique of Rudin is the technique that Zoli subsequently saw how to develop into a tremendously powerful method for constructing spaces, first his Q -set spaces, then his ZFC Dowker space, and so on.

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Title: Sequentially Linearly Lindelof spaces

A space X is *Sequentially Linearly Lindelöf* if for every set $A \subseteq X$ with $|A| = \lambda$, λ regular, uncountable and $\lambda \leq w(X)$ there is a subset $B \subseteq A$ so that $|A| = |B|$ and B converges to a point in X .

Problem. *Can one prove in ZFC alone the existence of a SLL space which is not Lindelöf?*

SLLnL spaces exist in every set-forcing extension of $V = L$ and in every model of set theory in which the singular cardinal hypothesis fails somewhere [1].

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Title: $C_p(X)$ and D-spaces

For definition and discussion of D-spaces see Arhangel'skii's contribution in this problem session.

Problem. *Let X be a compact space or a Lindelöf Σ -space. Must $C_p(X)$ be a D-space? a hereditarily D-space?*

Motivation: in 1980s Baturov proved that for $Y \subset C_p(X)$ where X is a Lindelöf Σ -space, $e(Y) = l(Y)$.

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Title: Characterizing countably rectifiable curves

Problem (1). *Characterize those metrizable continua which under some compatible metric are the union of countably many sets with finite linear Hausdorff measure.*

Problem (2). *Is it true that such a continuum may be embedded in R^3 so that the embedding is the union of countably many sets with finite linear Hausdorff measure with respect to the Euclidean metric?*

Reference Notes:

Eilenberg and Harrold characterized those continua which have finite linear Hausdorff measure. Fremlin showed such such continua may be embedded in R^3 to have finite linear measure.

Partial results and references may be found in "Continua with σ - finite linear measure," Measure Theory, Oberwolfach 1990, *Supplemento di Rendiconti del Circolo Matematico di Palermo, serie II*, no. 38, 1992.

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Title: A Linearly Lindelöf problem

Problem. *Is there a linearly Lindelöf non-Lindelöf normal space of weight \aleph_ω ?*

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Title: Commuting, coincidence-point-free maps on a triod

The following question is fairly popular among continuum theorists, but I think it deserves wider recognition.

Problem. *Let T denote a simple triod—three copies of the interval $[0, 1]$ with all the 0's identified. Does there exist a pair of continuous maps on T which commute and are coincidence-point free? I.e., do there exist continuous $f, g : T \rightarrow T$ such that $f \circ g = g \circ f$ and $f(x) \neq g(x)$ for all $x \in T$?*

If you could find such a pair of maps you would have a fantastic new continuum theory example. Let X denote the inverse limit of a sequence consisting of countably many triods, with every bonding map equal to f . Now use g to define a map $g^* : X \rightarrow X$ by:

$$g^*(\langle x_0, x_1, x_2, \dots \rangle) = \langle g(x_1), g(x_2), g(x_3), \dots \rangle.$$

Since $x_0 = f(x_1) \neq g(x_1)$, the map g^* is a fixed point free map defined on the tree-like continuum X . If you draw a diagram representing the above equation, you'll see that g^* is induced by using g as a “diagonal” mapping from the $(i + 1)$ st space back to the i th space—the result is a diagram of commuting parallelograms. It is worth mentioning that you can't define such a fixed point free map with a commuting diagram of squares, since T has the fixed point property.

On the other hand, a “no” answer might be a step in proving that triod-like continua (i.e., an inverse limit of a sequence of triods) have the fixed-point property. More importantly, it might be a step in answering the more general question “What does it mean, topologically, for a pair of continuous maps to commute?”

Now for a bit of history. David Bellamy described the first fixed point free mapping of a tree-like continuum. Shortly thereafter, Lex Oversteegen and Jim Rogers produced examples based on Bellamy's ideas which were described as inverse limits using a commuting parallelogram diagram like the one above. In these examples, the number of vertices in the factor spaces goes to infinity as $n \rightarrow \infty$ (as does as the complexity of the bonding and inducing maps). This makes it natural to ask whether there can be a “simplest” such example, with each factor space a triod and just two distinct maps. When I first heard of this question as a grad student, it was attributed to David Bellamy. However, he refuses to take credit for asking it. As far as I know, neither Lex nor Jim will take credit for asking the question either.

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Title: Does $w\Delta$ imply $\Sigma^\#$?

The title refers to two classes of “Generalized metric spaces” whose definitions are found in Gruenhage's Handbook article. The precise problem is:

Problem. *Is every regular $w\Delta$ -space $\Sigma^\#$?*

There is a class of generalized metrizable spaces called " β -spaces" that includes all $w\Delta$ -spaces, $\Sigma^\#$ -spaces, and semistratifiable spaces: see Gruenhagen's article again. In a way, the question is asking whether it is really redundant to be listing $w\Delta$ spaces along with the $\Sigma^\#$ -spaces.

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Title: **Normality and point-countable bases**

Problem. *Is there a collectionwise normal space with a point-countable base that is not paracompact?*

Amazingly enough, there do not even seem to be any consistency results on this in either direction. Still more amazingly, the same is true if we add "perfectly normal" to the conditions. In this form the problem was posed as a related problem in "Classic Problems" (see [1]).

If "collectionwise" is omitted, then all our current knowledge has to do with the questions of whether every [perfectly] normal space with a point-countable base is collectionwise normal. This is because every known normal space with a point countable base has the property that if it is collectionwise normal, it is also paracompact.

This second pair of questions sits snugly between the twin issues of whether every metacompact normal Moore space is metrizable and whether every first countable normal space is collectionwise normal. On the one hand, every normal Moore space is perfectly normal, every metacompact Moore space has a point-countable base, and every collectionwise normal Moore space is metrizable. On the other hand, every space with a point countable base is first countable.

No known model or axiom distinguishes between these two issues, which are inextricably intertwined with large cardinal axioms. Yes to both is consistent if it is consistent that there is a strongly compact cardinal; but there is a metacompact nonmetrizable normal Moore space if the covering lemma holds over the Core Model. See references [2] and [3] for more on what this means.

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Title: **How strong is "hereditarily strongly cwH "?**

Problem. *Is every hereditarily strongly collectionwise Hausdorff space normal? what if the space is regular? locally compact?*

A space is said to be strongly collectionwise Hausdorff (“strongly cwH”) if every closed discrete subspace expands to a discrete collection of open sets. These questions are inspired by some applications of large cardinals, including two old (1988) theorems of Zoltan Balogh, described in the abstract for my November talk; see also Reference [1]. Another application is the consistency (modulo a supercompact cardinal) of every hereditarily strongly cwH, locally compact, locally connected space being both (hereditarily) collectionwise normal and (hereditarily) countably paracompact: see Reference [3].

I am even unaware of a ZFC example of a hereditarily strongly cwH space that is not collectionwise normal. The only consistent locally compact example I know of is a normal manifold of Mary Ellen Rudin’s using \diamond^+ which is not collectionwise normal.

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Title: **Balogh’s own $P = NP$ problem**

Problem. *Given a class P of normal spaces, define NP to be the class of all spaces X such that $X \times Y$ is normal for all Y in P . Is there a class P of normal spaces for which $P = NP$?*

This problem was posed by Balogh at the 1995 Spring Topology Conference in Newark, Delaware. He carefully chose the notation for obvious reasons. The more general theme of what NP is for a given P is what was behind the three Morita conjectures, two of which were solved by Balogh after he posed his $P = NP$ problem. The Morita conjectures said, in Balogh’s notation, that $P = NNP$ for the classes of discrete spaces (for which NP is obviously the class of all normal spaces) and metrizable spaces, and they gave a characterization of NNP for the class of compact metrizable spaces.

The operator N induces a Galois correspondence: P is a subclass of NNP and if P is a subclass of Q then NQ is a subclass of NP . As is usual with all Galois correspondences, $NP = NNP$ for all P . It provides a whole general topic for problems: given a nice class P , can we find nice characterizations for NP and NNP ? Of course, once we have these three, adding more N ’s does not introduce new classes.

Thanks to Tamano’s theorem, if one begins with the class of compact spaces, then NP is the class of paracompact spaces, and it is also known that NNP is the class of sigma-locally compact paracompact spaces.

It also provides natural starting points for Balogh’s $P = NP$ problem. If P is a subclass of NP , as is clearly the case where P is the class of metrizable spaces, one can try adding spaces from NP to P one at a time in the hope that as P grows and NP shrinks, they come together in a solution to Balogh’s $P = NP$ problem. This simple approach will work only if all finite powers of the spaces we add to P are normal. In fact, any P which equals NP has to be closed under all finite products. So Balogh’s $P = NP$ problem is equivalent to:

Is there a class P of normal spaces such that every finite product of spaces in P is normal, and such that every normal space not in P either (1) has a non-normal product with some member of P or (2) has a non-normal finite power?

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Title: **Balogh's New Classic Problem**

PREFACE

This note has been adapted from Zoltan Balogh's contribution to *New Classic Problems* [2] and from the introduction in Balogh's article *On two problems concerning Baire sets in normal spaces* [1]. This note will appear in the collection *Problems from Topology Proceedings* [7].

A PROBLEM OF KATĚTOV

Given a topological space X , let $\text{Borel}(X)$ and $\text{Baire}(X)$ denote the σ -algebras generated by the families $\text{closed}(X) = \{F : F \text{ closed in } X\}$ and $\text{zero}(X) = \{F : F \text{ is a zero set in } X\}$, respectively. The following question is due, without the phrase “in ZFC”, to M. Katětov [6].

Problem (M. Katětov [6]). *Is there, in ZFC, a normal T_1 space X such that $\text{Borel}(X) = \text{Baire}(X)$ but X is not perfectly normal (i.e., $\text{closed}(X) \neq \text{zero}(X)$)?*

What if X is also locally compact? first countable? hereditarily normal?

Notes. There are several consistency examples given by Z. Balogh in [1]. CH implies that there is a locally compact locally countable X satisfying the conditions of the problem. The existence of a first countable, hereditarily paracompact X is consistent, too.

However, as summarized by the following theorem, a space giving a positive answer to the question cannot satisfy certain properties.

Theorem. *Let X be a normal T_1 space, and let A be a closed Baire subset of X . Then A is a zero set in X if one of the following conditions hold.*

- X is compact (P. R. Halmos [4]).
- X is paracompact and locally compact (W. W. Comfort [3]).
- X is submetacompact and locally compact (D. Burke).
- X is Lindelöf and Čech-complete (W. W. Comfort [3]).
- X is a subparacompact $P(\omega)$ -space (R. W. Hansell [5]).

In [1], Balogh gave a counterexample to the following related question of K. A. Ross and K. Stromberg. The construction makes use of the technique of E. K. van Douwen and H. H. Wicke [9] and W. Weiss [10]

Problem (K. A. Ross and K. Stromberg [8]). *If X is a normal locally compact Hausdorff space and A is a closed Baire set in X , is A a zero set?*

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Title: **Para-Lindelöf collectionwise normal spaces**

Problem. *Are para-Lindelöf collectionwise normal spaces paracompact?*

This was first asked by W. Fleissner and G. M. Reed [2]. It is Problem 109 from S. Watson's list [4]

Problem. *Are meta-Lindelöf, collectionwise normal space paracompact?*

This is R. Hodel's question [3] and also Problem 110 from Watson's list.

Zoltan Balogh [1] constructed a hereditarily collectionwise normal, hereditarily meta-Lindelöf, hereditarily realcompact Dowker space.

Balogh listed some open questions about meta-Lindelöf and para-Lindelöf Dowker spaces at the end of his article [1].

- (1) Is there a para-Lindelöf, collectionwise normal Dowker space?
- (2) Is there a para-Lindelöf Dowker space?
- (3) Is there a meta-Lindelöf, collectionwise normal and first countable Dowker space?
- (4) (D. Burke) Is there a meta-Lindelöf, collectionwise normal and countably paracompact space which is not paracompact?
- (5) Is there a first countable Dowker space in ZFC?

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Title: **Embedding free topological groups into unitary groups**

Problem. *Let $F(X)$ denote the (Markov or Graev) free topological group on a compact metric space X . Is it isomorphic with a topological subgroup of the group $U(\ell_2)$ of all unitary operators of the separable Hilbert space, equipped with the strong operator topology (that is, the topology induced from the usual Tychonoff power $(\ell_2)^{\ell_2}$)? What about the free topological group on the convergent sequence with the limit? In fact, the author of the present problem does not even know if the free topological group $F(X)$ on an arbitrary Tychonoff space X embeds into $U(\ell_2)$ as a topological subgroup.*

Embeddability of a topological group G into $U(\ell_2)$ with the strong topology is equivalent to the fact that continuous positive definite functions on G separate identity from closed subsets not containing it, cf. for instance the paper by Megrelishvili [1] and references therein.

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Title: **Bing points of countable-dimensional continua**

A space is countable-dimensional if it is the union of countably many finite-dimensional subsets. By a continuum we mean a metric compact connected space, and \dim stands for the dimension. A continuum X is hereditarily indecomposable, if for any two intersecting subcontinua K and L of X , either $K \subset L$ or $L \subset K$. If X is an infinite-dimensional continuum then by $B_\infty(X)$ we denote the set of Bing points of X , i.e. the set of points x such that every non-trivial subcontinuum of X containing x is infinite-dimensional.

Problem. *Does there exist for every integer $n \geq 2$ a countable-dimensional (if possible, hereditarily indecomposable) continuum X with $\dim B_\infty(X) \geq n$?*

For $n \in \{-1, 0, 1\}$, examples of countable-dimensional infinite-dimensional hereditarily indecomposable continua X with $\dim B_\infty(X) = n$ were constructed in [PR].

The positive answer to this problem would yield the negative answer to the following

Problem (R.Engelking and E.Pol [EP]). *Can every compact metrizable countable-dimensional space be mapped onto a finite-dimensional space by a mapping with finite-dimensional fibers?*

Indeed, assume that X_n is a countable-dimensional continuum such that $\dim B_\infty(X_n) \geq n$ and let $f : X_n \rightarrow Y$ be a mapping onto a continuum Y with finite-dimensional fibers. Then using a reasoning involving the monotone-light factorization of f , one checks that $\dim Y \geq n$. In effect, the one point compactification of the free union $\bigoplus_{n=2}^\infty X_n$ of the spaces X_n cannot be mapped onto a finite-dimensional space by a mapping with finite-dimensional fibers.

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Title: **Base-base, base-cover and base-family paracompactness**

Problem (1). *Is every subspace of the Sorgenfrey line base-base paracompact?*

A space is base-base paracompact (defined by John Porter in [9] under a different name) if it has an open base such that every subfamily which is still a base has a locally finite subcover. Although he was mainly interested in another property (base paracompactness, see also [10]), he observed in [9] that every base-base paracompact space is a D -space, and that all metric spaces, all non-archimedean spaces, and the Sorgenfrey line are base-base paracompact. (The above problem was implicit in [9] and was discussed by John Porter and other topologists in Auburn before he left for Murray State University. It is still unknown if every paracompact space is base-base paracompact [9].) In [11] we called a space base-cover paracompact (a stronger property) if it has an open base every subcover of which has a locally finite subcover, and proved that a subspace of the Sorgenfrey line is F_σ iff it is base-cover paracompact. It is known (including work of Zoltan Balogh) that Lusin subspaces of the Sorgenfrey line and, under MA, subspaces of cardinality less than continuum are Hurewicz [1], [4], [5], and therefore totally paracompact [3] (i.e. every base has a locally finite subcover).

The next problem was formulated by Gary Gruenhage after the author stated that base-family paracompactness is a generalized metrization property and solved some related problems. A space is base-family paracompact (defined in [11] under a different name) if it has an open base every subfamily of which contains a subfamily with the same union such that the latter subfamily is locally finite at each point of that union. Every proto-metrizable space (in particular every non-archimedean space) is base-family paracompact [11].

Problem (2). *Is every compact, perfectly normal, base-family paracompact space metrizable?*

There are consistent examples of perfectly normal, non-archimedean and non-metrizable spaces, e.g. the branch space of a Souslin tree, see [8], section 7 of [6], [12], [2]. Compact proto-metrizable spaces are metrizable [8]. Perfectly normal, non-metrizable compacta are discussed in [7]. The Alexandroff double arrow is not base-family paracompact; the one-point compactification of an uncountable discrete space is base-family paracompact but not perfectly normal [11].

Problem (3). *Is every base-family paracompact space monotonically normal?*

There are more questions related to these new classes of spaces in [9] and [11] (see also [10] and two forthcoming papers of the author about base-cover and base-family paracompactness).

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Title: **Some old favorites**

Problem (1). *If a normal T_2 space is the union of countably many open metrizable spaces, is it metrizable?*

I have shown the answer yes for $\text{card} < \mathfrak{c}$ under various set-theoretic assumptions. I also have a T_3 space example which is not a Moore space in ZFC.

Problem (2). *Does there exist a ZFC example of a T_3 perfect space which does not have a σ -discrete dense subset; in particular does there exist a first countable one?*

Call a space *star-compact* if each open cover \mathcal{U} has a finite subset H such that the set V of all elements in \mathcal{U} which have non-empty intersection with an element of H is also a cover.

Problem (3).

- a) *Is each star-compact Moore space countably compact?*
- b) *Does there exist a first countable, star-compact T_3 space that is not countably compact?*

The answer to both a) and b) is yes under CH.

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Z. Balogh, G. Gruenhagen, *Base multiplicity in generalized compact spaces*, Topology and Appl., 115 (2001), pp. 139-151.

Accepted:

Z. Balogh, *Dowker spaces and their constructions*, Encyclopedia of General Topology, North-Holland Publishing Company.

Z. Balogh, G. Gruenhagen, *On a problem of Nagata*, Topology and Applications.

Z. Balogh, *Reflecting point-countable families*, Proc. Amer. Math. Soc.

Z. Balogh, *Locally nice spaces and Axiom R*, Topology and Appl.

Z. Balogh, *Elementary submodels and covering properties*, Topology Proc.

Z. Balogh and J. Griesmer, *On the multiplicity of jigsawed bases in compact and countably compact spaces*, Topology and Applications.

Z. Balogh, *On density and number of points in somewhat Lindelof spaces*, Topology Proceedings.

Work in progress:

Z. Balogh, *A Lindelof Q-set space of cardinality \mathfrak{c}* , handwritten notes.

Z. Balogh, *Two theorems on the number of points*, handwritten notes.

CONFERENCE PRESENTATIONS – SIX YEARS

On normal, screenable spaces, 1996 Spring Topology Conference, Ball State University, Muncie, IN, March 1996.

A normal Screenable space in ZFC, 30-minute invited talk, Eighth Prague Topological Symposium, August 1996.

On Realcompact Dowker Spaces, Tennessee Topological Conference, Nashville, TN, June 1996.

A Lindelof Q-set of cardinality \mathfrak{c} , 20 minute invited talk, Special session on set-theoretic topology, Amer. Math. Soc. Meeting #915, University of Tennessee, Chattanooga, TN, October 1996.

Elementary submodels and covering properties, 60-minute plenary talk, 1997 spring topology Conference, Lafayette, La, April 1997.

Strongly almost disjoint sets and weakly uniform bases, 20 minute invited talk, special session on set-theoretic techniques in topology and analysis, AMS meeting in Atlanta, GA, October 1997.

Countable in countable bases in compact and compact-like spaces, 20 minute invited talk, Third Joint Meeting of the Amer. Math. Soc. And the Sociedad Matematica Mexicana, December 1997.

Base multiplicity, presented at the 1998 Spring Topology Conference held at George Mason University, March 1998.

Set-theoretic constructions of covers in ZFC, 13th Summer Conference on General Topology and Applications, UNAM de Mexico, Mexico City, June 1998.

A survey of Dowker spaces, Invited keynote address, International Colloquium on Topology, Janos Bolyai Society, Gyula, Hungary, August 1998.

Strict p-spaces via elementary submodels, 20 minute invited talk, Special session on the 926th Amer. Math. Soc. Meeting held at Wake Forest University, Winston-Salem, North Carolina.

On Collectionwise Normal Metalindelof spaces, 1999 Spring Topology Conference, March 18-20, 1999, University of Utah.

Special Session on Set Theory at the Erdos and his Mathematics conference, Budapest, July 4-11, 1999, two one-hour invited talks entitled "A Basic Dowker Space"

- On Shrinking Properties*, Fourteenth Summer Conference on Topology and Applications, August 4-7, 1999, C. Post Campus of Long Island University.
- International Conference on Topology and its Applications, August 23-27, 1999, Kanagawa University, Yokohama, Japan. Title of one-hour plenary address: *Dowker Spaces and Covering Properties*
- Nonshrinking open covers and K. Morita's duality conjectures*, Spring Topology Conference, San Antonio, TX, March 16-19, 2000.
- Reflections on reflecting non-metacompactness*, TOPO2000 Summer Conference on Topology and its Applications, Oxford, OH (!) July 26-29, 2000.
- Elementary submodels and zapping through a lot sets*, one hour plenary address, First Turkish International Conference on Topology and Applications, Istanbul, August 2-5, 2000.
- Covering properties and elementary submodels*, 20 minute invited talk, Special Session on Set Theory and Set-theoretic topology, AMS Fall Central Section Meeting, Toronto, Canada, September 23-24, 2000.
- Locally nice spaces and Axiom R*, 20 minute invited talk, Special session on set-theoretic topology and Boolean algebra, AMS at the University of Kansas, Lawrence, Kansas, March 30-31, 2001.
- Reflecting point-countable open families*, 16th Summer Conference on Topology and Applications, New York, July 18-21, 2001.
- Reflection Theorems and elementary submodels*, Invited talk, 9th Prague Topological Symposium, Prague, August 19-23, 2001.
- Invited speaker for the conference: TOPOLOGY in Matsue; International Conference on Topology and its Applications, Shimane University in Matsue City, Japan, on June 24-28, 2002.

GRANTS

- (1) Investigations on Normal and Paracompact Spaces, National Science Foundation, DMS-9623391, June 15 1996-May 31, 1999, \$68,100
- (2) Investigations on Normal and Paracompact Spaces, National Science Foundation, DMS-9977099, August 1, 1999-July 31, 2002 (covers July 1999), \$74,340

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