Zoltán Balogh Memorial Topology Conference
Abstracts

Metrizably Fibered GO-spaces

Harold R Bennett, *Texas Tech University*

We characterize GO-spaces that are metrizably fibered in terms of certain quotient spaces and in sequences of open covers.

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Forcing hereditarily separable countably compact group topologies

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Using forcing we produce a model of $\mathsf{ZFC} + \mathsf{CH}$ (with $2^{\mathfrak{c}}$ arbitrarily large) and, in this model, we obtain a characterization of the Abelian groups G (necessarily of size at most $2^{\mathfrak{c}}$) which admit:

(i) a hereditarily separable group topology,
(ii) a group topology making G into an S-space,
(iii) a hereditarily separable group topology that is either precompact, or pseudocompact, or countably compact (and which can be made to contain no non-trivial convergent sequences),
(iv) a hereditarily separable connected and locally connected group topology that is either pseudocompact, or countably compact (and which can be made to contain no non-trivial convergent sequences),
(v) a group topology making G into an S-space that is either precompact, or pseudocompact, or countably compact (and which also can be made without non-trivial convergent sequences, if necessary).

As a by-product, we completely describe the algebraic structure of the Abelian groups of size at most $2^{\mathfrak{c}}$ which admit, at least consistently, a countably compact group topology (possibly without non-trivial convergent sequences).

We obtain also a complete solution to a 1980 problem of van Douwen about the cofinality of the size of countably compact Abelian groups (a counter-examle to van Douwen's conjecture was obtained recently by Tomita).

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Radially Compact spaces

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We introduce a new class of spaces which we show to be compact and pseudoradial. We also show that any finite product of these spaces remain in the class. We ask if this is an approach to establishing that a finite product of compact pseudoradial spaces is again pseudoradial.

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Proper forcing revisited

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One of the mysteries of iterated forcing theory is that there seems to be no good generalization of properness to the context of iterations with uncountable support. There are ZFC theorems (mostly due to Shelah) that show many natural conjectures are false. We will present a version of properness that does work for forcing with uncountable supports, and give an example of how it can be used.

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Coarser Connected Topologies

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Let P be a separation property from this list: Hausdorff, Urysohn, regular, normal, (κ-)collectionwise normal, (κ-)collectionwise Hausdorff, metric. Let (X, τ) be a propery P space with a strongly separated, closed discrete set C, $|C| = d(X) \geq \mathfrak{c}$. Then there is a coarser topology σ such that (X, σ) is a connected property P space.

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The mathematics of Zoltan T. Balogh

Gary Gruenhage, *Auburn University*

I will give an overview of Zoli's work, including a list of what I consider his six "greatest hits".

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Calibers and Tightness

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We strengthen results of Shapirovskii and of Archangelskii, respectively by proving that

(1) if a T_3 space X is the union of ω_1 compact subspaces of countable tightness and ω_1 is a caliber of X then X is separable;
(2) if a T_3 space X has no uncountable free sequences and ω_1 is a caliber of X then X has a dense subset of size at most continuum.

Under CH, (2) implies that any Lindelöf T_3 space of countable tightness is separable if it has ω_1 as a caliber. However, generic left separated spaces yield (consistent) examples which show that the bound continuum given by (2) [or Archangelskii's original result] for the density is sharp. We also present a (consistent) counterexample to a problem of Okunev and Tkachuk, namely a compact T_2 space such that ω_1 is a caliber of every dense subspace but it has uncountable d-tightness.

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Hereditarily normal compactifications of metrizable spaces

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Hereditarily normal compactifications of metrizable spaces Abstract: We show that the non-separable metrizable hedgehog-space $J(\omega_1)$ cannot be embedded in any hereditarily normal $\aleph_1$-compact space. In particular, $J(\omega_1)$ has no monotonically normal compactification; this answers a question made by P. Gartside. There also exist strongly paracompact metrizable spaces without monotonically normal compactifications; this follows from the result that a metrizable space X is locally separable if the product space $X \times \mathbb{I}$ has a hereditarily normal $\aleph_1$-compactification.

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On semi-θ-generalized closed sets

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The aim of this paper is to introduce and study the concept of semi-θ-generalized closed sets in topological spaces via the semi-θ-closure operator which was introduced by G. Di Maio and T. Noiri [2]. Semi-R1, $T_{3/4}$ and gs-regular spaces are characterized. The concepts of Tsθg-spaces and semi-θ-generalized continuous functions are introduced.

A subset A of a space X is called a *Semi-θ-generalized closed set* (denoted by sθg-closed) if sClt(A)is a subset of G whenever A is a subset of G and G is open in X. The class of all sθg-closed subsets of X is denoted by SθGC(X). The complement of each sθg-closed subsets of X is called sθg-open and the class of all sθg-open subsets of X is denoted by SθGO(X). A function $f : X \to Y$ is called semi-θ-generalized continuous (sθg-continuous) if the inverse image of each closed subset of Y is semi-θ-generalized closed. Equivalently, f is semi-θ-generalized continuous if the inverse image of each open subset of Y is semi-θ-generalized open.

The following lemmas show that the class of SθGC(X) is properly placed between the class of θ-generalized closed sets and generalized semi-closed sets and the class of semi-θ-generalized continuous functions is properly placed between the class of gs-continuous functions and θ-g-continuous functions

- (i) Each semi-θ-closed set in X is sθg-closed.
- (ii) Each θ-generalized closed subset of X is semi-θ-generalized closed.
- (iii) Each semi-θ-generalized continuous function is gs-continuous.
- (iv) Each θ-g-continuous function is semi-θ-generalized continuous

Also we give some characterizations of semi-R1 and $T_{3/4}$-spaces.

- (i) A topological space X is semi-R1 if and only if sCl{x}=sClθ{x} for each x in X.
- (ii) A topological space X is $T_{3/4}$-space if and only if each semi-θ-generalized closed set is semi-θ-closed.
- (iii) A topological space X is $T_{3/4}$-space if and only if for each semi-θ-generalized closed F in X and each x in F, there exist disjoint semi-open sets U and V containing x and F respectively.

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κ-Fréchet-Urysohn Spaces

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(Arhangel'skii, 1999) A Hausdorff topological space X is called *κ-Fréchet-Urysohn* if for every open subset A of X and every $x \in \overline{A}$ there exists a sequence of points of A converging to x. We will discuss the properties of κ-Fréchet-Urysohn spaces and several results including: The regular topological space X is κ-Fréchet-Urysohn if and only if X is a κ-pseudo-open image of a metric space.

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Trees and lines

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Given linear orderings for its nodes, a tree $(T, \leq)$ gives rise to lines (= linearly ordered sets) in two very different ways, namely by branch space constructions and by lexicographic orderings of the tree itself. Without further restrictions, any line is order isomorphic to the branch space of some tree, and also to some lexicographically ordered tree. However, with reasonable restrictions on the tree, the situation is quite different.

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Fat-Psi constructions

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We survey some old and some new results obtained by means of fat-Psi constructions.

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Balogh's 'lost' theorems on paracompactness in locally compact spaces

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In the 1988 volume of AMS Abstracts, Zoltán Balogh announced two striking applications of large cardinals to the theme of when a locally compact space is paracompact. Both applications involved the concept of "strongly ω_1-cwH." This is the property that every closed discrete subspace of cardinality $\leq \omega_1$ expands to a discrete collection of open sets. If "discrete" is weakened to "disjoint," we get the more familiar concept of "ω_1-cwH."

The first announced application used Fleissner's Axiom R; the second used Martin's Maximum. Application 1: Every locally compact, $\delta\theta$-refinable, strongly ω_1-cwH space is paracompact. Application 2: Every locally compact, hereditarily strongly ω_1-cwH space is either paracompact or contains a perfect preimage of ω_1.

The second application has finally appeared in print. It is in a paper by Zoltán titled "Locally nice spaces and Axiom R," which appeared a few days before the conference: *Topology and its Applications* 125 (2002)

335–341. The fragment of Martin's Maximum actually used is MA(ω_1) + Axiom R. It has several other striking results which will be recounted in this talk along with some strengthenings due to Alan Dow and the speaker.

The first application is still shrouded in mystery. I do not even know whether it is valid. With "strongly cwH" in place of "strongly ω_1-cwH," we have a ZFC theorem, also due to Zoli, and appearing already in "Paracompactness in locally Lindelof spaces", Canad. J. Math. 38(1986), 719-727. I will show how Application 1 is valid for countably tight spaces.

A ZFC example of a locally compact, metacompact, screenable, hereditarily cwH non-normal space with a countable T_0-separating open cover will also be presented, showing just how essential the word "strongly" is in Zoli's theorems.

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Base-cover and base-family paracompactness

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We present results from the author's dissertation (Auburn University, August 2002). A space is base-cover paracompact is it has an open base every subcover of which has a locally finite subcover. A space is base-family paracompact if it has an open base every subfamily of which has a subfamily with the same union and which is locally finite at each point from its union. Several open questions are posed too.

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An α-normal non-β-normal Moore space

John E. Porter, *Murray State University*

The notions of α- normal and β-normal spaces were introduced by A. Arhangel'skii and L. Ludwig. A. Arhangel'skii and L. Ludwig showed that if $2^\omega < 2^{\omega_1}$ then separable, α-normal Moore spaces are metrizable. It is shown that ff there exists a separable, α-normal, non-metrizable Moore space then there exists a Q-set. Alos, M. Wage's example of a collectionwise Hausdorff, non-metrizable Moore space is shown to be collectionwise α-normal but not β-normal.

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Computational models of topological spaces

Mike Reed, *Oxford University Computing Laboratory*

Two major questions in domain theory are:

Can each complete Moore space (complete metrizble space) be realized as the maximal elements of a Scott domain?

The author shows that the answer is negative for complete Moore spaces and provides a positive (partial) result for metrizable spaces.

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A joint paper

Mary Ellen Rudin, *University of Wisconsin*

Zoli and I were attracted by similar ideas and often even by the same problem. I gloried in his solutions, but once we wrote a joint paper!

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FUF and boundedly FUF spaces

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The class of FUF-spaces (recently introduced by Reznichenko and Sipacheva) is a subclass of Frechet-Urysohn spaces. A space is FUF if for every point x and every family F of finite subsets of X, if F forms a local π-net at x, then F has a subset that converges to x.

We investigate the relationship of FUF to other classes of Frechet-Urysohn spaces including the α_i-spaces of Arhangel'skii.

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Optimal constant weight binary codes

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Let X be an n-set and $\binom{X}{w}$the collection of all w-subsets of X equipped with the Hamming distance. A subset of $\binom{X}{w}$ is called a constant weight code and its members are codewords. The distance in a given code means the minimum distace between a pair of distinct codewords. The problem of determining the optimal size of a constant weight code having a preassigned distance in it is discussed.

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Riffs on Zoli's "Locally nice spaces under Martin's axiom"

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In the cited paper, Balogh extended Szentmiklóssy's "no compact S-spaces" theorem to get that, under $\mathsf{MA} + \sim\mathsf{CH}$,

(B) *Locally countable subspaces of size $< \mathfrak{c}$ are σ-discrete in compact spaces of countable tightness.*

Balogh used this to obtain a large number of interesting consequences. In joint work with Paul Larson, we prove that (B) is consistent with:

(C) *Every normal first countable space is collectionwise Hausdorff.*

This enables us to obtain a variety of consistency results along the same lines as Balogh's but more powerful. Our results depend on work of Todorcevic which currently requires supercompact cardinals, but that requirement can probably be eliminated. Assuming then the consistency of a supercompact cardinal, we obtain the consistency of

(i) *locally compact, locally hereditarily Lindelöf, hereditarily normal spaces are paracompact if and only if they do not include a perfect pre-image of ω_1*,

(ii) *locally compact perfectly normal spaces are paracompact*,

(iii) *locally compact hereditarily normal spaces with G_δ-diagonals are metrizable*,

(iv) *locally compact perfectly normal spaces of size $\aleph_1$ are metrizable*,

(v) *locally compact spaces with hereditarily normal squares are metrizable.*

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The Urysohn Property and two step iteration of almost disjoint families

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Let $\mathcal{A}_0$ be an infinite maximal almost disjoint family of infinte subsets of the natural numbers ω, and $\psi(\mathcal{A}_0) = \omega \cup \mathcal{A}_0$ with the topology in which for every natural number $n \in \omega$, the singletons $\{n\}$ are isolated for $n \in \omega$ and each $A \in \mathcal{A}_0$ has a local base consisting of sets of the form $\{A\} \cup A \setminus F$ where F is a finite subset of ω. This is a well-known space in topology. We consider a collectionwise type separation property on $\psi(\mathcal{A}_0)$ which can be equivalently stated as the Urysohn separation property on an extension of $\psi(\mathcal{A}_0)$. The extension is defined from $\psi(\mathcal{A}_0)$ by taking $\mathcal{A}_1$, an infinite, maximal almost disjoint family of countable subsets of the non-isolated points of $\psi(\mathcal{A}_0)$, and defining neighborhood of an elements $X \in \mathcal{A}_1$ by $\{X\} \cup (X \setminus G) \cup (\cup\{A \setminus F(A) : A \in X \setminus G\})$ where G is finite, and $F(A)$ is finite for all $A \in X \setminus G$. The resulting space is denoted $\psi(\mathcal{A}_0, \mathcal{A}_1)$ It is consistent that there exists $\mathcal{A}_0, \mathcal{A}_1$ such that $\psi(\mathcal{A}_0, \mathcal{A}_1)$ is Urysohn. We will prove in ZFC that there exists a maximal $\mathcal{A}_0$ such that for every maximal $\mathcal{A}_1$, $\psi(\mathcal{A}_0, \mathcal{A}_1)$ is not Urysohn.

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Monotone Extensions

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We characterize the closed subsets H of a monotonically normal space X that allow an extender $e : C(H) \to C(X)$ such that

(1) If f is a continuous function from H into the reals, ef is a continuous extension of f.
(2) If f and g are in $C(H)$ with $f(x) \leq g(x)$ for all $x \in H$, then $f(x) \leq g(x)$ for all $x \in X$.

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